Improving the angular resolution of the conical Wolter-I silicon pore optics (SPO) mirror design for the International X-ray Observatory (IXO)

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ABSTRACT

The mirror design for the International X-ray Observatory (IXO) is currently following two paths: a segmented slumped glass shell Wolter-I design, and a Silicon Pore Optics (SPO) conical approximation to the Wolter-I design. The conical approximation used for the SPO imposes a lower limit to the angular resolution which puts this option at a potential disadvantage. In this paper we describe ways in which this can be circumvented. We analyse the surface profile modifications that can be made to lift this limitation and show that a much closer approximation to the Wolter I ideal is possible. We describe several ways in which a much tighter angular resolution limit could be achieved in practice and discuss ways in which this can be implemented in the manufacture of the SPO.

Keywords: X-ray optics for astronomy

1. INTRODUCTION

For the next generation of X-ray telescopes we need a technology that can provide large collecting areas of several square meters at ∼1 keV and an angular resolution of a few arc seconds or better. The baseline technology for the IXO primary mirror within the ESA Cosmic Visions Program is SPO as described by Beijersbergen et al.,[1] an example of which is shown in Fig. 1. As originally conceived the SPO stacks form a conical approximation to the classical 2-reflection Wolter I geometry because the Silicon plates in the stacks are curved in the azimuthal direction but the axial profiles are linear. Under ideal conditions, without any additional figure errors, each pore produces a very narrow line focus with length equal to the radial pore size and the full Point Spread Function (PSF) is the summation of the contributions from millions of pores across the aperture at all azimuthal angles. The PSF can be described analytically as a power law radial surface brightness profile with index -1 and an

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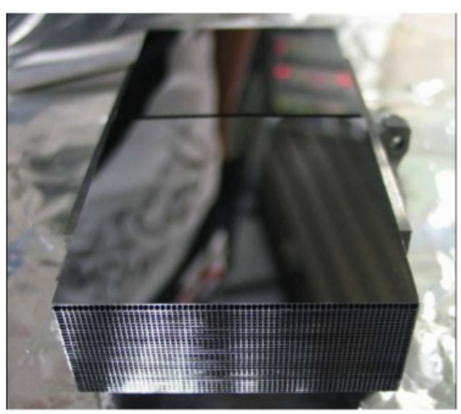

Figure 1. A conical approximation W-I module made of stacked Silicon plates.

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abrupt truncation at radius $d/2$ where d is the radial pore size. Using $d = 605$ microns, a focal length of 20 m and an aperture with inner radius 0.25 m and outer radius 1.9 m, which is the current baseline, the ideal PSF has a Half Energy Width (HEW) of $d/2 = 303$ microns equivalent to 3.13 arc seconds. This is illustrated using the ray tracing results shown in Fig. 2. The surface brightness profile from the ray tracing is shown in Fig. 3.

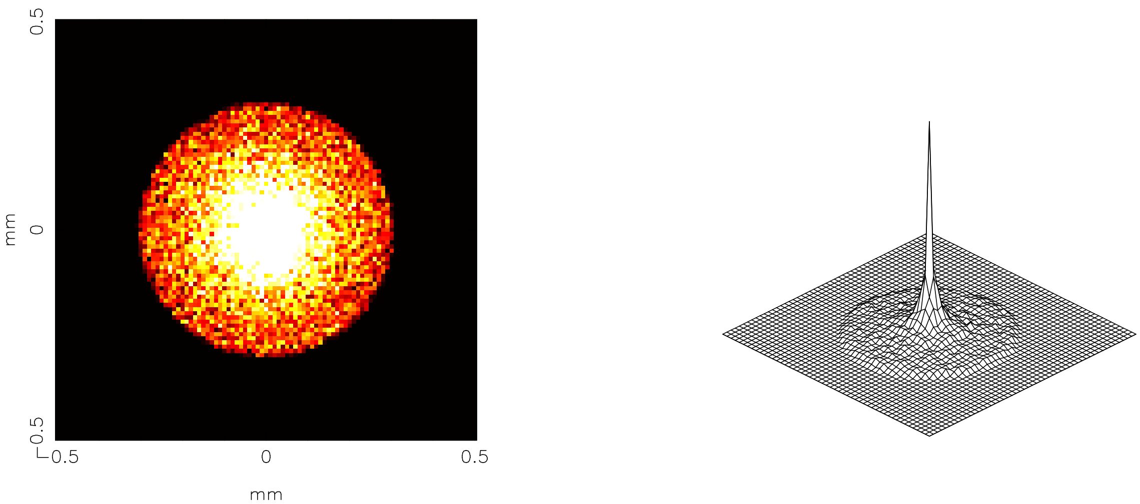

Figure 2. The full aperture PSF for a perfect conical approximation Si Pore Optic IXO mirror.

The power law decay is as expected and the cut-off at ≈ 0.3 mm is obvious. The HEW estimated from the ray tracing is 3.14 arc seconds in agreement with the analytical value within the limitations of the binning used to calculate the surface brightness profile. Given that the required angular resolution for IXO is 5 arc seconds HEW at 1 keV the allocation of over 3 arc seconds of this to the conical geometry imposes rather severe constraints on the rest of the error budget for the SPO mirror design. The geometric limit on angular resolution imposed by the pore size and the conical approximation could be reduced using smaller pores, i.e. smaller d. However at a wavelength of ~ 10 $\mathring{A}$ (X-ray energy ~ 1 keV) diffraction starts to dominate if $d < 0.25$ mm (see Spaan[2]) and the manufacture of smaller Si pores presents practical difficulties. Furthermore, unless the conical approximation limit can be lifted SPOs will never offer the possibility of higher angular resolutions in the future. Below we consider what is required to remove the conical approximation from the SPO design.

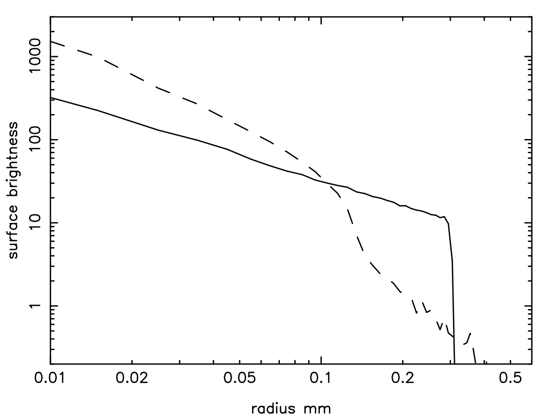

Figure 3. Surface brightness of the PSF estimated by ray tracing. Conical approximation solid curve. Constant axial curvature design dashed curve.

2. THE REFLECTING SURFACE EQUATIONS FOR THE CONICAL APPROXIMATION SPO

The conventional origin datum for defining the Wolter I equations is the position of the on-axis focus.[3] A more convenient datum for the SPO surfaces is the intersection of the optical axis with the join plane between the 1st and 2nd surface stacks. If x is the axial coordinate then the 1st surface pores are at positive x and the 2nd surface pores are at negative x. In the following we only consider designs in which the grazing angles on the 1st and 2nd surfaces are equal or more strictly almost equal. This gives the maximum efficiency when we include the X-ray reflectivities but has no effect on the angular resolution which can be achieved. If the radial width of the pores is d and the length of the pores is L then we set the ratio to be $d/L = \tan(\theta_g) \approx \theta_g$ where θ_g is the grazing angle of reflection for axial rays. This ensures that the full reflecting surface of each pore is illuminated for an on-axis source and maximises the aperture utilization of the pores. If a particular pair of pores meet in the join plane at radius R in the aperture then the grazing angle is given by

$$\theta_g = \frac{1}{4}\arctan\left(\frac{R + d/2}{F}\right) \tag{1}$$

where F is the focal length, the axial distance from the join plane to the on-axis focus. Note that the radius is increased a little by $d/2$. This ensures that rays which intersect the reflection surface at the centre of the pore are brought to the correct focus at a distance F from the join plane. Using the above $L \approx 4dF/R$ such that the length of the pores varies inversely with radius if d is fixed. This is required to provide the maximum aperture utilization as noted above.

The equation for the axial profile of the 1st surface in the conical approximation is $r_1 = \tan(\theta_g)x + R$. Writing this in the more conventional form for the conic section generators we have

$$r_1^2 = \tan^2(\theta_g)x^2 + 2\tan(\theta_g)Rx + R^2 \tag{2}$$

and similarly we can define the 2nd surface which has a cone angle 3 times that of the 1st surface as

$$r_2^2 = \tan^2(3\theta_g)x^2 + 2\tan(3\theta_g)Rx + R^2 \tag{3}$$

The conical axial profiles defined by Equations 2 and 3 are plotted as the dashed lines in Fig. 4 for a typical pore at a radius $R = 1$ m in the IXO aperture.

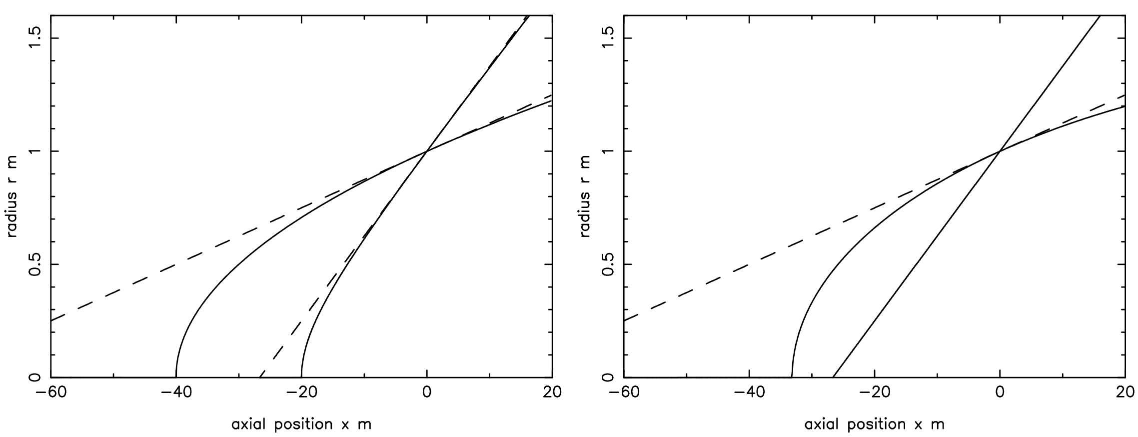

Figure 4. Left-hand panel: The conical approximation axial profiles, dashed lines, and equal curvature axial profiles, solid lines. Right-hand panel: a double curvature 1st surface axial profile with a conical approximation 2nd surface.

3. CORRECTING THE CONICAL APPROXIMATION ABERRATION

In order to improve the angular resolution of the SPO we need to introduce some curvature into the axial profiles of the pores. The full width of the conical approximation PSF shown in Figs. 2 and 3 is equal to the radial pore size, d, and to eliminate this spreading we need curvature along the length L of both the 1st and 2nd surfaces. X-rays at the outer edges of the PSF are reflected from close to the ends of the pores and these must be deflected by $d/(2F)$ radians to bring them into focus. This can be achieved by increasing the grazing angles at the ends of the pores by $d/(8F)$ radians. The ends of the pores are an axial distance of $L/2$ from the pore centre so the axial curvature required is given by

$$\frac{d^2r}{dx^2} = -\frac{d}{4FL} = -\frac{\tan(\theta_g)}{4F} \approx -\frac{R}{16F^2} \tag{4}$$

This curvature can be included using a extra term $\alpha(x - L/2)^2$ in Equation 2 and $\alpha(x + L/2)^2$ in Equation 3 where

$$\alpha = R\frac{d^2r}{dx^2} = -\frac{R\tan(\theta_g)}{4F} = -\tan^2(\theta_g) \tag{5}$$

The presence of the offset axial positions $x - L/2$ and $x + L/2$ ensures that the gradients at the centre of the pore axial profiles are the same as in the original conical approximation. The axial profiles are then

$$r_1^2 = \tan^2(\theta_g)x^2 + 2\tan(\theta_g)Rx + R^2 - \tan^2(\theta_g)(x - L/2)^2 \tag{6}$$

$$r_2^2 = \tan^2(3\theta_g)x^2 + 2\tan(3\theta_g)Rx + R^2 - \tan^2(\theta_g)(x + L/2)^2 \tag{7}$$

We have dubbed these the *equal curvature axial profiles* and they are plotted as the solid curves in the left-hand panel of Fig. 4. These profiles are not the same as the true Wolter I parabola-hyperbola but they are very close, especially in the vicinity of the join plane. Ray tracing the full IXO aperture using the equal curvature axial profiles for all the pores gives a HEW of $\approx 1/100$ arc seconds on-axis with a small axial shift of 40 microns for the optimum focal position. This high performance is possible because the pores are sub-mm in size and the maximum axial length of the Si plates is 200 mm. If we increase the pore size by a factor of 10 to 6 mm and the maximum axial length is increased to 2 m then the conical approximation limits the HEW to ~ 30 arc seconds but the equal curvature profiles still provide a HEW of 7/100 arc seconds with a modest shift of 0.47 mm of the focal plane. The equal curvature axial profiles, Equations 6 and 7, offer a near perfect limiting performance which is independent of the radial pore size.

It is not necessary to add curvature to both the 1st and 2nd reflecting surfaces. We can, for example, double the curvature of the 1st surface using $\alpha = -2\tan^2(\theta_g)$ and leave the 2nd surface as a simple cone. This configuration is shown in the right-hand panel of Fig. 4 and, remarkably. it gives an on-axis HEW of 3/100 arc seconds.

The curvature introduced above is proportional to the radius R. We can still get a significant improvement over the conical approximation is we employ a constant axial curvature, independent of R, for all pores across the aperture. For example, if we use the double curvature design but fix the curvature to the value at $R = 1$ m, -1.56×10^{-7}, then $\alpha = -1.56 \times 10^{-7}R$ and the HEW is 0.85 arc seconds. The PSF is illustrated in Fig. 5 and the surface brightness profile is shown in Fig. 3. Comparison of this PSF with the conical approximation PSF also shown in Fig. 3 indicates how the constant curvature design has significantly suppressed the extended wings of the distribution.

4. SPO STACKS WITH AXIAL CURVATURE

The optimum axial curvature required, as described above, gives a sagittal sag of $\Delta r = dL/32F = d^2/(8R)$. For the baseline IXO value of $d = 0.605$ mm this gives $\Delta r = 0.18$ microns for the inner stacks at $R = 0.25$ m where $L = 194$ mm and $\Delta r = 0.024$ microns for the outer stacks at $R = 1.9$ m where $L = 26$ mm. These sag values are equivalent to a constant axial slope change of $\Delta\theta = d/(16F)$ at the ends of the pores compared to the conical approximation design across all the pores in the aperture. Using the baseline pore size and focal length this is 0.39 arc seconds. i.e. the Δr sag values or $\Delta\theta$ gradient values required are very small for all positions in the aperture and for all the axial stack lengths required.

Of course, in order to capitalise on the potential improvements in angular resolution offered by the addition of axial curvature we have to build Si stacks to a very high accuracy using a shaping mandrel that incorporates such

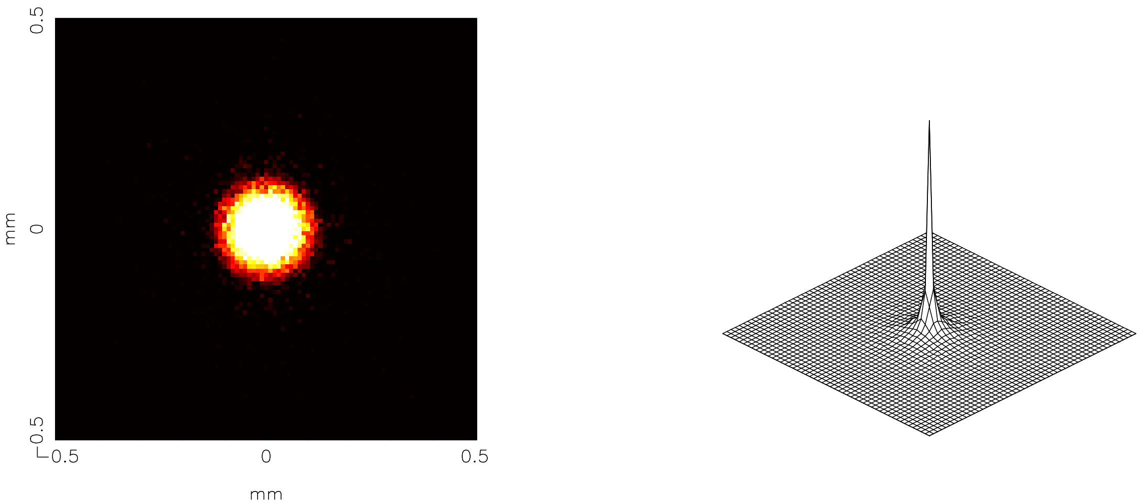

Figure 5. The full aperture PSF of the constant curvature design.

curvature. However, it makes no sense to try and produce a conical approximation mandrel and then allocate the appropriate HEW of 3.13 arc seconds associated with this poor approximation to the required figure. We suggest a better approach is to attempt to produce stacking mandrels with the required axial curvature and allocate ~ 3 arc seconds HEW to the problem of manufacturing the mandrels and manufacturing stacks which follow the curvature of those mandrels. If we can control the gradient errors at the ends of the pores to better than ≈ 0.4 arc seconds during the stacking process then we will produce a stack with a performance better than the conical approximation.

Alternatively, we could attempt to introduce a constant axial curvature of -1.56×10^{-7} equivalent to $\alpha = -1.56 \times 10^{-7} R$ in all the mandrels and hence all the stacks. If this approximation could be achieved then the limiting HEW would be ≈ 0.85 arc seconds which is a factor of 3.7 better than the original conical form. The constant curvature design has the attraction that the axial curvature is the same for all mandrels irrespective of the azimuthal curvature radius R. This might make the manufacture of the mandrels and metrology of the figure of the Si plates during stacking easier.

The successful adoption of any axial curvature scheme in the SPO stacks requires that the Si plates can be bent to the right figure and bonding of the stacks is not compromised. Given the very small curvature needed this is very unlikely. Metrology of existing stacks taken during manufacture indicates that some of the plates do in fact have a negative axial curvature of the correct order. The problem is undoubtedly not one of producing and bonding the curvature into the stacks but of controlling the curvature to within the tolerances indicated above.

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